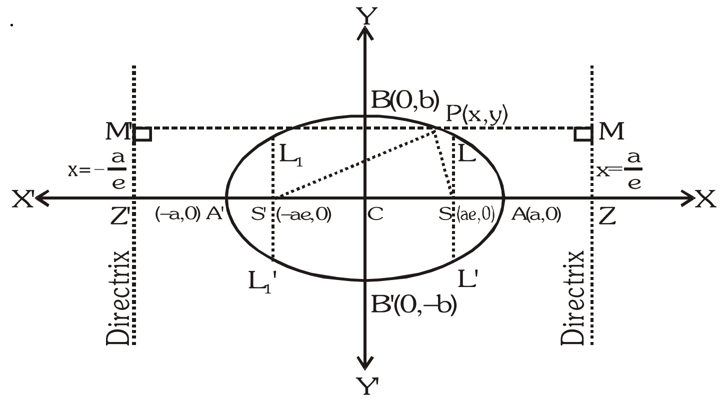
ELLIPSE

1. STANDARD EQUATION & DEFINITION :

Standard equation of an ellipse referred to its principal axes along the co-ordinate axes is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. where $a > b$ & $b^2 = a^2(1 - e^2) \Rightarrow a^2 - b^2 = a^2 e^2$.

where e = eccentricity $(0 < e < 1)$.

FOCI : $S \equiv (ae, 0)$ & $S' \equiv (-ae, 0)$.



(a) **Equation of directrices** :

$$x = \frac{a}{e} \quad \& \quad x = -\frac{a}{e}.$$

(b) **Vertices** :

$A' \equiv (-a, 0)$ & $A \equiv (a, 0)$.

(c) **Major axis** : The line segment A' A in which the foci S' & S lie is of length 2a & is called the **major axis** $(a > b)$ of the ellipse. Point of intersection of major axis with directrix is called **the foot of the directrix** (z) $\left(\pm\frac{a}{e}, 0\right)$.

(d) **Minor Axis** : The y-axis intersects the ellipse in the points $B' \equiv (0,-b)$ & $B \equiv (0, b)$. The line segment B'B of length 2b $(b < a)$ is called the **Minor Axis** of the ellipse.

(e) **Principal Axes** : The major & minor axis together are called **Principal Axes** of the ellipse.

(f) **Centre** : The point which bisects every chord of the conic drawn through it is called the **centre** of the conic. $C \equiv (0,0)$ the origin is the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(g) **Diameter** : A chord of the conic which passes through the centre is called a **diameter** of the conic.

(h) **Focal Chord** : A chord which passes through a focus is called a **focal chord.**

(i) **Double Ordinate** : A chord perpendicular to the major axis is called a **double ordinate.**

(j) **Latus Rectum** : The focal chord perpendicular to the major axis is called the **latus rectum**.

(i) Length of latus rectum (LL') $=\frac{2b^2}{a}=\frac{(\text{minor axis})^2}{\text{major axis}}=2a(1-e^2)$

(ii) Equation of latus rectum : $x = \pm ae$.

(iii) Ends of the latus rectum are $L\left(ae, \frac{b^2}{a}\right), L'\left(ae, -\frac{b^2}{a}\right), L_1\left(-ae, \frac{b^2}{a}\right)$ and $L_1'\left(-ae, -\frac{b^2}{a}\right)$.

(k) **Focal radii** : $SP = a - ex$ & $S'P = a + ex$ $\Rightarrow$ **SP + S'P = 2a = Major axis.**

(l) **Eccentricity** : $e = \sqrt{1-\frac{b^2}{a^2}}$

Note :

(i) The sum of the focal distances of any point on the ellipse is equal to the major Axis. Hence distance of focus from the extremity of a minor axis is equal to semi major axis. **i.e BS = CA.**

(ii) If the equation of the ellipse is given as $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & nothing is mentioned, then the rule is to assume that $a > b$.

Illustration 1 : If LR of an ellipse is half of its minor axis, then its eccentricity is -

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{2}}{3}$

Solution : As given $\frac{2b^2}{a} = b$ $\Rightarrow$ $2b = a$ $\Rightarrow$ $4b^2 = a^2$

$\Rightarrow$ $4a^2(1 - e^2) = a^2$ $\Rightarrow$ $1 - e^2 = 1/4$

$\therefore$ $e = \sqrt{3}/2$ **Ans. (C)**

Illustration 2 : Find the equation of the ellipse whose foci are (2, 3), (–2, 3) and whose semi minor axis is of length $\sqrt{5}$.

Solution : Here S is (2, 3) & S' is (–2, 3) and $b = \sqrt{5}$ $\Rightarrow$ $SS' = 4 = 2ae \Rightarrow ae = 2$

but $b^2 = a^2(1 - e^2)$ $\Rightarrow$ $5 = a^2 - 4$ $\Rightarrow$ $a = 3$.

Hence the equation to major axis is y = 3

Centre of ellipse is midpoint of SS' i.e. (0, 3)

$\therefore$ Equation to ellipse is $\frac{x^2}{a^2} + \frac{(y-3)^2}{b^2} = 1$ or $\frac{x^2}{9} + \frac{(y-3)^2}{5} = 1$ **Ans.**

Illustration 3 : Find the equation of the ellipse having centre at (1, 2), one focus at (6, 2) and passing through the point (4, 6).

Solution : With centre at (1, 2), the equation of the ellipse is $\frac{(x-1)^2}{a^2} + \frac{(y-2)^2}{b^2} = 1$. It passes through the point (4, 6)

$\Rightarrow$ $\frac{9}{a^2} + \frac{16}{b^2} = 1$ .... (i)

Distance between the focus and the centre = (6 – 1) = 5 = ae

$\Rightarrow$ $b^2 = a^2 - a^2e^2 = a^2 - 25$ .... (ii)

Solving for a^2 and b^2 from the equations (i) and (ii), we get $a^2 = 45$ and $b^2 = 20$.

Hence the equation of the ellipse is $\frac{(x-1)^2}{45} + \frac{(y-2)^2}{20} = 1$ **Ans.**

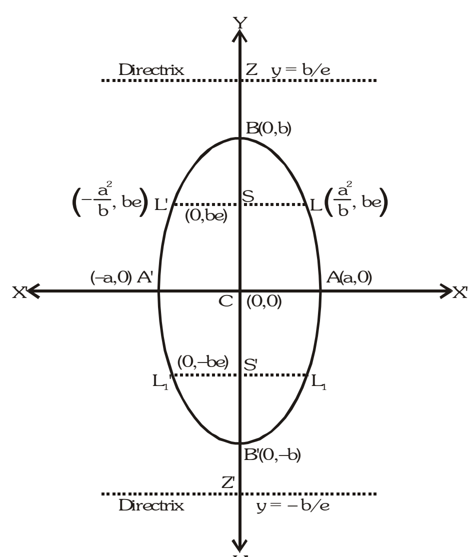
Do yourself - 1 :

(i) If LR of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a < b) is half of its major axis, then find its eccentricity.

(ii) Find the equation of the ellipse whose foci are (4, 6) & (16, 6) and whose semi-minor axis is 4.

(iii) Find the eccentricity, foci and the length of the latus-rectum of the ellipse $x^2 + 4y^2 + 8y - 2x + 1 = 0$.

2. ANOTHER FORM OF ELLIPSE : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a <b)



(a) AA' = Minor axis = 2a

(b) BB' = Major axis = 2b

(c) $a^2 = b^2(1 - e^2)$

(d) Latus rectum $LL' = L_1L_1' = \frac{2a^2}{b}$, equation $y = \pm be$

(e) Ends of the latus rectum are :

$$L\left(\frac{a^2}{b}, be\right), L'\left(-\frac{a^2}{b}, be\right), L_1\left(\frac{a^2}{b}, -be\right), L_1'\left(-\frac{a^2}{b}, -be\right)$$

(f) Equation of directrix $y = \pm b/e$

(g) Eccentricity : $e = \sqrt{1 - \frac{a^2}{b^2}}$

Illustration 4 : The equation of the ellipse with respect to coordinate axes whose minor axis is equal to the distance between its foci and whose LR = 10, will be-

(A) $2x^2 + y^2 = 100$ (B) $x^2 + 2y^2 = 100$ (C) $2x^2 + 3y^2 = 80$ (D) none of these

Solution : When a > b

As given $2b = 2ae \Rightarrow b = ae$ ...... (i)

Also $\frac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$ ...... (ii)

Now since $b^2 = a^2 - a^2e^2 \Rightarrow b^2 = a^2 - b^2$ [From (i)]

$\Rightarrow 2b^2 = a^2$ ...... (iii)

(ii), (iii) $\Rightarrow a^2 = 100,\ b^2 = 50$

Hence equation of the ellipse will be $\frac{x^2}{100} + \frac{y^2}{50} = 1 \Rightarrow x^2 + 2y^2 = 100$

Similarly when $a < b$ then required ellipse is $2x^2 + y^2 = 100$ **Ans. (A, B)**

Do yourself - 2 :

(i) The foci of an ellipse are (0, ±2) and its eccentricity is $\frac{1}{\sqrt{2}}$. Find its equation

(ii) Find the centre, the length of the axes, eccentricity and the foci of ellipse $12x^2 + 4y^2 + 24x - 16y + 25 = 0$

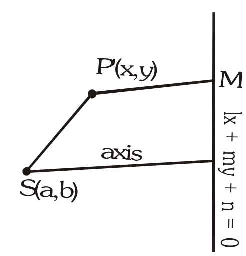
(iii) The equation $\frac{x^2}{8-t} + \frac{y^2}{t-4} = 1$, will represent an ellipse if

(A) $t \in (1, 5)$ (B) $t \in (2, 8)$ (C) $t \in (4, 8) - \{6\}$ (D) $t \in (4, 10) - \{6\}$

3. GENERAL EQUATION OF AN ELLIPSE

Let (a, b) be the focus S, and $lx + my + n = 0$ is the equation of directrix.

Let P(x, y) be any point on the ellipse. Then by definition.



$\Rightarrow$ **SP = e PM** (e is the eccentricity) $\Rightarrow (x-a)^2 + (y-b)^2 = e^2\frac{(lx+my+n)^2}{(l^2+m^2)}$

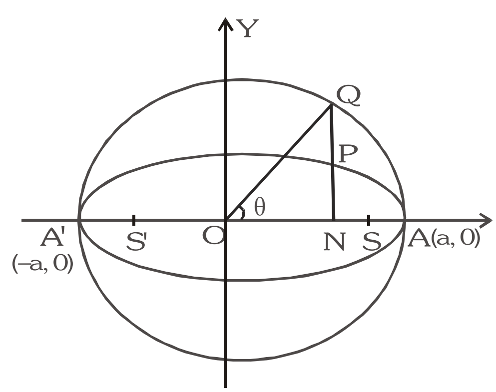
$\Rightarrow (l^2 + m^2)\{(x-a)^2 + (y-b)^2\} = e^2\{lx + my + n\}^2$

4. POSITION OF A POINT W.R.T. AN ELLIPSE :

The point $\mathbf{P(x_1, y_1)}$ lies **outside**, **inside** or **on** the ellipse according as ; $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$ > < or = 0.

5. AUXILLIARY CIRCLE/ECCENTRIC ANGLE :

A circle described on major axis as diameter is called the **auxiliary circle.** Let Q be a point on the auxiliary circle $x^2 + y^2 = a^2$ such that QP produced is perpendicular to the x-axis then P & Q are called as the CORRESPONDING POINTS on the ellipse & the auxiliary circle respectively.'θ' is called the **ECCENTRIC ANGLE** of the point P on the ellipse $(0 \le \theta < 2\pi)$.



Note that $\frac{l(PN)}{l(QN)} = \frac{b}{a} = \frac{\text{Semi minor axis}}{\text{Semi major axis}}$

Hence "If from each point of a circle perpendiculars are drawn upon a fixed diameter then the locus of the points dividing these perpendiculars in a given ratio is an ellipse of which the given circle is the auxiliary circle".

6. PARAMETRIC REPRESENTATION :

The equations $x = a\cos\theta$ & $y = b\sin\theta$ together represent the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

where θ is a parameter (eccentric angle).

Note that if $P(\theta) \equiv (a\cos\theta, b\sin\theta)$ is on the ellipse then ; $Q(\theta) \equiv (a\cos\theta, a\sin\theta)$ is on the auxiliary circle.

7. LINE AND AN ELLIPSE :

The line $y = mx + c$ meets the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in two points real, coincident or imaginary according as c^2 is < = or > $a^2m^2 + b^2$.

Hence $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $c^2 = a^2m^2 + b^2$.

The equation to the chord of the ellipse joining two points with eccentric angles α & β is given by $\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}$.

Illustration 5 : For what value of λ does the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$.

Solution : $\because$ Equation of ellipse is $9x^2 + 16y^2 = 144$ or $\frac{x^2}{16}+\frac{y^2}{9}=1$

Comparing this with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then we get $a^2 = 16$ and $b^2 = 9$

and comparing the line $y = x + \lambda$ with $y = mx + c$ $\therefore$ $m = 1$ and $c = \lambda$

If the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$, then $c^2 = a^2m^2 + b^2$

$\Rightarrow$ $\lambda^2 = 16\ \ 1^2 + 9$ $\Rightarrow$ $\lambda^2 = 25$ $\therefore$ $\lambda = \pm 5$ **Ans.**

Illustration 6 : If α, β are eccentric angles of end points of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then $\tan\alpha/2.\ \tan\beta/2$ is equal to -

(A) $\frac{e-1}{e+1}$ (B) $\frac{1-e}{1+e}$ (C) $\frac{e+1}{e-1}$ (D) $\frac{e-1}{e+1}$

Solution : Equation of line joining points 'α' and 'β' is $\frac{x}{a}\cos\frac{\alpha+\beta}{2}+\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}$

If it is a focal chord, then it passes through focus (ae, 0), so $e\cos\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}$

$$\Rightarrow \frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}=\frac{e}{1} \quad \Rightarrow \quad \frac{\cos\frac{\alpha-\beta}{2}-\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}+\cos\frac{\alpha+\beta}{2}}=\frac{e-1}{e+1}$$

$$\Rightarrow \frac{2\sin\alpha/2\ \ \sin\beta/2}{2\cos\alpha/2\ \ \cos\beta/2}=\frac{e-1}{e+1} \quad \Rightarrow \quad \tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{e-1}{e+1}$$

using (–ae, 0) , we get $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{e+1}{e-1}$ **Ans. (A,C)**

Do yourself - 3 :

(i) Find the position of the point (4, 3) relative to the ellipse $2x^2 + 9y^2 = 113$.

(ii) A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, (a > b) having slope –1 intersects the axis of x & y in point A & B respectively. If O is the origin then find the area of triangle OAB.

(iii) Find the condition for the line x cosθ + ysinθ = P to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

8. TANGENT TO THE ELLIPSE $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$:

(a) **Point form :** Equation of tangent to the given ellipse at its point (x_1, y_1) is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

Note : For general ellipse replace x^2 by (xx_1), y^2 by (yy_1), 2x by $(x + x_1)$, 2y by $(y + y_1)$, 2xy by $(xy_1 + yx_1)$ & c by (c).

(b) **Slope form :** Equation of tangent to the given ellipse whose slope is 'm', is $y = mx \pm \sqrt{a^2m^2+b^2}$

Point of contact are $\left(\frac{\mp a^2m}{\sqrt{a^2m^2+b^2}}, \frac{\pm b^2}{\sqrt{a^2m^2+b^2}}\right)$

Note that there are two tangents to the ellipse having the same m, i.e. there are two tangents parallel to any given direction.

(c) **Parametric form :** Equation of tangent to the given ellipse at its point (a cos θ, b sin θ), is

$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$$

Note :

(i) The eccentric angles of point of contact of two parallel tangents differ by π.

(ii) Point of intersection of the tangents at the point α & β is $\left(a\frac{\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, b\frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}\right)$

Illustration 7 : Find the equations of the tangents to the ellipse $3x^2 + 4y^2 = 12$ which are perpendicular to the line y + 2x = 4.

Solution : Let m be the slope of the tangent, since the tangent is perpendicular to the line y + 2x = 4.

∴ mx – 2 = –1 ⇒ $m = \frac{1}{2}$

Since $3x^2 + 4y^2 = 12$ or $\frac{x^2}{4}+\frac{y^2}{3}=1$

Comparing this with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

∴ $a^2 = 4$ and $b^2 = 3$

So the equation of the tangent are $y=\frac{1}{2}x\pm\sqrt{4\times\frac{1}{4}+3}$

⇒ $y = \frac{1}{2}x \pm 2$ or x – 2y ± 4 = 0. **Ans.**

Illustration 8 : The tangent at a point P on an ellipse intersects the major axis in T and N is the foot of the perpendicular from P to the same axis. Show that the circle drawn on NT as diameter intersects the auxiliary circle orthogonally.

Solution : Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Let P(acosθ, bsinθ) be a point on the ellipse. The equation of the tangent at P is $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$. It meets the major axis at T ≡ (a secθ, 0).

The coordinates of N are (a cosθ, 0). The equation of the circle with NT as its diameter is $(x - a\sec\theta)(x - a\cos\theta) + y^2 = 0$.

⇒ $x^2 + y^2 - ax(\sec\theta + \cos\theta) + a^2 = 0$

It cuts the auxiliary circle $x^2 + y^2 - a^2 = 0$ orthogonally if

$2g \cdot 0 + 2f \cdot 0 = a^2 - a^2 = 0$, which is true. **Ans.**

Do yourself - 4 :

(i) Find the equation of the tangents to the ellipse $9x^2 + 16y^2 = 144$ which are parallel to the line $x + 3y + k = 0$.

(ii) Find the equation of the tangent to the ellipse $7x^2 + 8y^2 = 100$ at the point (2, –3).

9. NORMAL TO THE ELLIPSE $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$:

(a) **Point form** : Equation of the normal to the given ellipse at (x_1, y_1) is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}= a^2 - b^2 = a^2e^2$.

(b) **Slope form** : Equation of a normal to the given ellipse whose slope is 'm' is $y = mx \mp \frac{(a^2-b^2)m}{\sqrt{a^2+b^2m^2}}$.

(c) **Parametric form** : Equation of the normal to the given ellipse at the point **(acosθ, bsinθ)** is **$ax\sec\theta - by\,\mathrm{cosec}\,\theta = (a^2 - b^2)$**.

Illustration 9 : Find the condition that the line ℓx + my = n may be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Solution : Equation of normal to the given ellipse at (a cos θ, b sin θ) is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$ ...(i)

If the line ℓx + my = n is also normal to the ellipse then there must be a value of θ for which line (i) and line ℓx + my = n are identical. For that value of θ we have

$$\frac{\ell}{\left(\frac{a}{\cos\theta}\right)}=\frac{m}{-\left(\frac{b}{\sin\theta}\right)}=\frac{n}{(a^2-b^2)} \quad \text{or} \quad \cos\theta=\frac{an}{\ell(a^2-b^2)} \qquad \text{...... (iii)}$$

and $$\sin\theta=\frac{-bn}{m(a^2-b^2)} \qquad \text{...... (iv)}$$

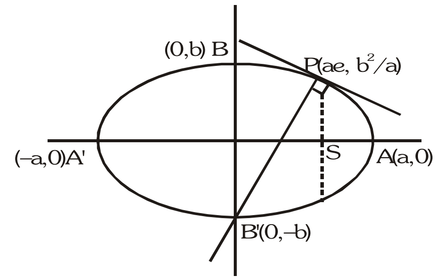
Squaring and adding (iii) and (iv), we get $1=\frac{n^2}{(a^2-b^2)^2}\left(\frac{a^2}{\ell^2}+\frac{b^2}{m^2}\right)$ which is the required condition.

Illustration 10 : If the normal at an end of a latus-rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through one extremity of the minor axis, show that the eccentricity of the ellipse is given by $e=\sqrt{\frac{\sqrt{5}-1}{2}}$

Solution : The co-ordinates of an end of the latus-rectum are (ae, b^2/a).

The equation of normal at P(ae, b^2/a) is

$$\frac{a^2x}{ae} - \frac{b^2(y)}{b^2/a} = a^2 - b^2 \quad \text{or} \quad \frac{ax}{e} - ay = a^2 - b^2$$



It passes through one extremity of the minor axis whose co-ordinates are (0, –b)

$\therefore$ $0 + ab = a^2 - b^2$ $\Rightarrow$ $(a^2b^2) = (a^2 - b^2)^2$

$\Rightarrow$ $a^2.a^2(1 - e^2) = (a^2 e^2)^2$ $\Rightarrow$ $1 - e^2 = e^4$

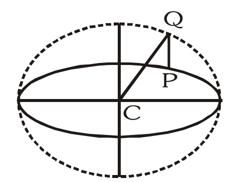
$\Rightarrow$ $e^4 + e^2 - 1 = 0$ $\Rightarrow$ $(e^2)^2 + e^2 - 1 = 0$

$\therefore$ $e^2 = \frac{-1 \pm \sqrt{1+4}}{2}$ $\Rightarrow$ $e = \sqrt{\frac{\sqrt{5}-1}{2}}$ (taking positive sign) **Ans.**

Illustration 11 : P and Q are corresponding points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the auxiliary circles respectively. The normal at P to the ellipse meets CQ in R, where C is the centre of the ellipse. Prove that CR = a + b

Solution : Let P ≡ (acosθ, bsinθ)

$\therefore$ Q ≡ (acosθ, asinθ)



Equation of normal at P is

$(a\sec\theta)x - (b\,\text{cosec}\theta)y = a^2 - b^2$ .... (i)

equation of CQ is $y = \tan\theta \,.\, x$ .... (ii)

Solving equation (i) & (ii), we get $(a - b)x = (a^2 - b^2)\cos\theta$

$x = (a + b)\cos\theta$, & $y = (a + b)\sin\theta$

$\therefore$ R ≡ ((a + b)cosθ, (a + b)sinθ)

$\therefore$ CR = a + b **Ans.**

Do yourself - 5 :

(i) Find the equation of the normal to the ellipse $9x^2 + 16y^2 = 288$ at the point (4, 3)

(ii) Let P be a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci F_1 and F_2. If A is the area of the triangle PF_1F_2, then find maximum value of A.

(iii) If the normal at the point P(θ) to the ellipse $\frac{x^2}{3} + \frac{y^2}{2} = 1$ intersects it again at the point Q(2θ), then find cosθ.

(iv) Show that for all real values of 't' the line $2tx + y\sqrt{1-t^2} = 1$ touches a fixed ellipse. Find the eccentricity of the ellipse.

10. CHORD OF CONTACT :

If PA and PB be the tangents from point $P(x_1,y_1)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

The equation of the chord of contact AB is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ or T = 0 (at x_1,y_1).

Illustration 12 : If tangents to the parabola $y^2 = 4ax$ intersect the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at A and B, the find the locus of point of intersection of tangents at A and B.

Solution : Let P ≡ (h, k) be the point of intersection of tangents at A & B

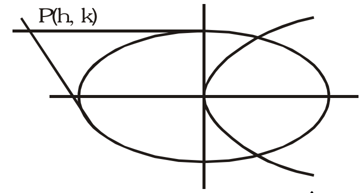
$\therefore$ equation of chord of contact AB is $\frac{xh}{a^2} + \frac{yk}{b^2} = 1$ .... (i)

which touches the parabola.

Equation of tangent to parabola $y^2 = 4ax$ is $y = mx + \frac{a}{m}$

$\Rightarrow \quad mx - y = -\frac{a}{m}$ .... (ii)

equation (i) & (ii) as must be same



$\therefore \quad \frac{m}{\left(\frac{h}{a^2}\right)} = \frac{-1}{\left(\frac{k}{b^2}\right)} = \frac{-\frac{a}{m}}{1} \Rightarrow \quad m = -\frac{h}{k}\frac{b^2}{a^2}$ & $m = \frac{ak}{b^2}$

$\therefore \quad -\frac{hb^2}{ka^2} = \frac{ak}{b^2} \Rightarrow$ locus of P is $y^2 = -\frac{b^4}{a^3}.x$ **Ans.**

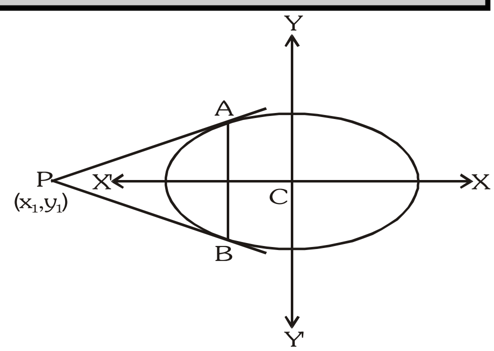
Do yourself - 6 :

(i) Find the equation of chord of contact to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at the point (1, 3).

(ii) If the chord of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are at right angles, then find $\frac{x_1x_2}{y_1y_2}$.

(iii) If a line $3x - y = 2$ intersects ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ at points A & B, then find co-ordinates of point of intersection of tangents at points A & B.

11. PAIR OF TANGENTS :



If $P(x_1,y_1)$ be any point lies outside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,
and a pair of tangents PA, PB can be drawn to it from P.
Then the equation of pair of tangents of PA and PB is $SS_1 = T^2$

where $S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$, $T = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$

i.e. $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1\right) = \left(\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1\right)^2$

12. DIRECTOR CIRCLE :

Locus of the point of intersection of the tangents which meet at right angles is called the **Director Circle.** The equation to this locus is $\mathbf{x^2 + y^2 = a^2 + b^2}$ i.e. a circle whose centre is the centre of the ellipse & whose radius is the length of the line joining the ends of the major & minor axis.

Illustration 13 : A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. Prove that the tangents at P and Q of the ellipse $x^2 + 2y^2 = 6$ are at right angles.

Solution : Given ellipse are $\frac{x^2}{4} + \frac{y^2}{1} = 1$ .... (i)

and, $\frac{x^2}{6} + \frac{y^2}{3} = 1$ .... (ii)

any tangent to (i) is $\frac{x\cos\theta}{2} + \frac{y\sin\theta}{1} = 1$ .... (iii)

It cuts (ii) at P and Q, and suppose tangent at P and Q meet at (h, k) Then equation of chord of contact of (h, k) with respect to ellipse (ii) is $\frac{hx}{6} + \frac{ky}{3} = 1$ .... (iv) comparing (iii) and (iv), we get $\frac{\cos\theta}{h/3} = \frac{\sin\theta}{k/3} = 1 \Rightarrow \cos\theta = \frac{h}{3}$ and $\sin\theta = \frac{k}{3} \Rightarrow h^2 + k^2 = 9$

locus of the point (h, k) is $x^2 + y^2 = 9 \Rightarrow x^2 + y^2 = 6 + 3 = a^2 + b^2$

i.e. director circle of second ellipse. Hence the tangents are at right angles.

13. EQUATION OF CHORD WITH MID POINT (x_1, y_1) :

The equation of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose mid-point be (x_1, y_1) is $T = S_1$

where $T = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$, $S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1$, i.e. $\left(\frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1\right) = \left(\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1\right)$

Illustration 14 : Find the locus of the mid-point of focal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

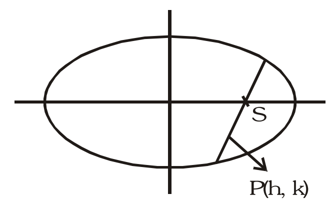
Solution : Let $P \equiv (h, k)$ be the mid-point

$\therefore$ equation of chord whose mid-point is given $\frac{xh}{a^2} + \frac{yk}{b^2} - 1 = \frac{h^2}{a^2} + \frac{k^2}{b^2} - 1$

since it is a focal chord,

$\therefore$ It passes through focus, either (ae, 0) or (–ae, 0)

If it passes through (ae, 0)



$\therefore$ locus is $\frac{ex}{a} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

If it passes through (–ae, 0)

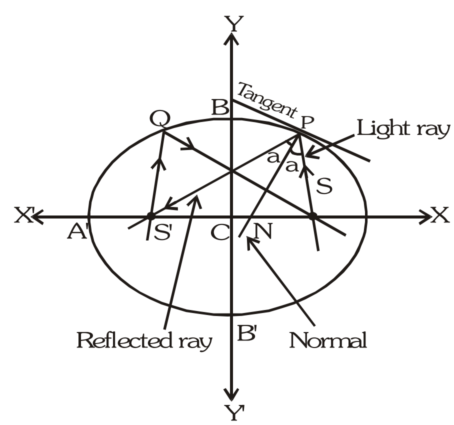
$\therefore$ locus is $-\frac{ex}{a} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ **Ans.**

Do yourself - 7 :

(i) Find the equation of chord of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ whose mid point be (–1, 1).

14. IMPORTANT POINTS :

Referring to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



(a) If P be any point on the ellipse with S & S' as its foci then $\ell(SP) + \ell(S'P) = 2a$.

(b) The tangent & normal at a point P on the ellipse bisect the external & internal angles between the focal distances of P. This refers to the well known reflection property of the ellipse which states that rays from one focus are reflected through other focus & vice versa .

(c) **The product of the length's of the perpendicular segments from the foci on any tangent to the ellipse is b^2** and the feet of these perpendiculars lie on its auxiliary circle and the tangents at these feet to the auxiliary circle meet on the ordinate of P and that the locus of their point of intersection is a similar ellipse as that of the original one.

(d) The portion of the tangent to an ellipse between the point of contact & the directrix subtends a **right angle** at the corresponding focus.

(e) If the normal at any point P on the ellipse with centre C meet the major & minor axes in G & g respectively, & if CF be perpendicular upon this normal, then

(i) PF . PG = b^2 (ii) PF . Pg = a^2

(iii) PG . Pg = SP . S' P (iv) CG . CT = CS^2

(v) locus of the mid point of Gg is another ellipse having the same eccentricity as that of the original ellipse.

[where S and S' are the focii of the ellipse and T is the point where tangent at P meet the major axis]

(f) Atmost four normals & two tangents can be drawn from any point to an ellipse.

(g) The circle on any focal distance as diameter touches the auxiliary circle.

(h) Perpendiculars from the centre upon all chords which join the ends of any perpendicular diameters of the ellipse are of constant length.

(i) If the tangent at the point P of a standard ellipse meets the axes in T and t and CY is the perpendicular on it from the centre then,

(i) Tt . PY = $a^2 - b^2$ and (ii) least value of Tt is a + b.

Do yourself - 8 :

(i) A man running round a racecourse note that the sum of the distance of two flag-posts from him is always 20 meters and distance between the flag-posts is 16 meters. Find the area of the path be encloses in square meters

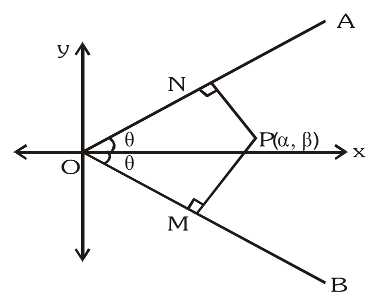
(ii) If chord of contact of the tangent drawn from the point (α, β) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches the circle $x^2 + y^2 = k^2$, then find the locus of the point (α, β).

Miscellaneous Illustration :

Illustration 15 : A point moves so that the sum of the squares of its distances from two intersecting straight lines is constant. Prove that its locus is an ellipse.

Solution : Let two intersecting lines OA and OB, intersect at origin O and let both lines OA and OB makes equal angles with x axis.

i.e., $\angle XOA = \angle XOB = \theta$.



$\therefore$ Equations of straight lines OA and OB are

$y = x\tan\theta$ and $y = -x\tan\theta$

or $x\sin\theta - y\cos\theta = 0$ .... (i)

and $x\sin\theta + y\cos\theta = 0$ .... (ii)

Let $P(\alpha, \beta)$ is the point whose locus is to be determine.

According to the example $(PM)^2 + (PN)^2 = 2\lambda^2$ (say)

$\therefore$ $(\alpha\sin\theta + \beta\cos\theta)^2 + (\alpha\sin\theta - \beta\cos\theta)^2 = 2\lambda^2 \Rightarrow 2\alpha^2\sin^2\theta + 2\beta^2\cos^2\theta = 2\lambda^2$

or $\alpha^2\sin^2\theta + \beta^2\cos^2\theta = \lambda^2 \Rightarrow \frac{\alpha^2}{\lambda^2\mathrm{cosec}^2\theta}+\frac{\beta^2}{\lambda^2\sec^2\theta}=1 \Rightarrow \frac{\alpha^2}{(\lambda\ \mathrm{cosec}\ \theta)^2}+\frac{\beta^2}{(\lambda\sec\theta)^2}=1$

Hence required locus is $\frac{x^2}{(\lambda\ \mathrm{cosec}\ \theta)^2}+\frac{y^2}{(\lambda\sec\theta)^2}=1$ **Ans.**

Illustration 16 : Find the condition on 'a' and 'b' for which two distinct chords of the ellipse $\frac{x^2}{2a^2}+\frac{y^2}{2b^2}=1$ passing through (a, –b) are bisected by the line x + y = b.

Solution : Let (t, b − t) be a point on the line x + y = b.

Then equation of chord whose mid point (t, b − t) is

$$\frac{tx}{2a^2}+\frac{y(b-t)}{2b^2}-1=\frac{t^2}{2a^2}+\frac{(b-t)^2}{2b^2}-1 \quad \text{.......... (i)}$$

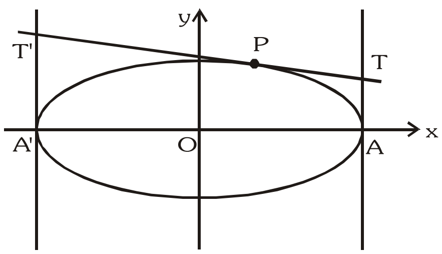
(a, –b) lies on (i) then $\frac{ta}{2a^2}-\frac{b(b-t)}{2b^2}=\frac{t^2}{2a^2}+\frac{(b-t)^2}{2b^2}$ $\Rightarrow$ $t^2(a^2+b^2)-ab(3a+b)t+2a^2b^2=0$

Since t is real $B^2 - 4AC \geq 0$ $\Rightarrow$ $a^2b^2(3a+b)^2 - 4(a^2+b^2)2a^2b^2 \geq 0$

$\Rightarrow$ $a^2 + 6ab - 7b^2 \geq 0$ $\Rightarrow$ $a^2 + 6ab \geq 7b^2$, which is the required condition.

Illustration 17 : Any tangent to an ellipse is cut by the tangents at the ends of the major axis in T and T'. Prove that circle on TT' as diameter passes through foci.

Solution : Let ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



and let P(acosϕ, bsinϕ) be any point on this ellipse

∴ Equation of tangent at P(acosϕ, bsinϕ) is

$$\frac{x}{a}\cos\phi+\frac{y}{b}\sin\phi=1 \quad \text{.....(i)}$$

The two tangents drawn at the ends of the major axis are x = a and x = –a

Solving (i) and x = a we get $T=\left\{a,\frac{b(1-\cos\phi)}{\sin\phi}\right\}\equiv\left\{a, b\tan\left(\frac{\phi}{2}\right)\right\}$

and solving (i) and x = – a we get $T'=\left\{-a,\frac{b(1+\cos\phi)}{\sin\phi}\right\}\equiv\left\{-a, b\cot\left(\frac{\phi}{2}\right)\right\}$

Equation of circle on TT' as diameter is $(x-a)(x+a)+(y-b\tan(\phi/2))(y-b\cot(\phi/2))=0$

or $x^2+y^2-by(\tan(\phi/2)+\cot(\phi/2))-a^2+b^2=0$ .... (ii)

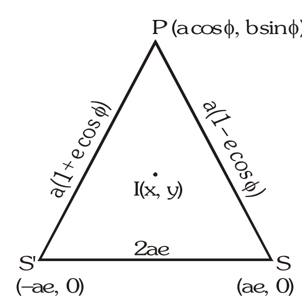
Now put x = ± ae and y = 0 in LHS of (ii), we get

$a^2e^2+0-0-a^2+b^2=a^2-b^2-a^2+b^2=0=\text{RHS}$

Hence foci lie on this circle

Illustration 18 : A variable point P on an ellipse of eccentricity e, is joined to its foci S, S'. Prove that the locus of the incentre of the triangle PSS' is an ellipse whose eccentricity is $\sqrt{\left(\frac{2e}{1+e}\right)}$.

Solution : Let the given ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



Let the co-ordinates of P are (a cosϕ, b sinϕ)

By hypothesis

$b^2 = a^2(1-e^2)$ and S(ae, 0), S'(–ae, 0)

∴ SP = focal distance of the point P = a – ae cos ϕ

and S'P = a + ae cos ϕ

Also SS' = 2ae

If (x, y) be the incentre of the ΔPSS' then

$$\therefore \quad x = \frac{(2ae)a\cos\phi + a(1-e\cos\phi)(-ae) + a(1+e\cos\phi)ae}{2ae + a(1-e\cos\phi) + a(1+e\cos\phi)}$$

$$x = ae\cos\phi \qquad \text{....... (i)}$$

$$y = \frac{2ae(b\sin\phi) + a(1+e\cos\phi).0 + a(1-e\cos\phi).0}{2ae + a(1-e\cos\phi) + a(1+e\cos\phi)}$$

$$\Rightarrow \quad y = \frac{eb\sin\phi}{(e+1)} \qquad \text{....... (ii)}$$

Eliminating ϕ from equations (i) and (ii), we get $\frac{x^2}{a^2e^2} + \frac{y^2}{\left[\frac{be}{e+1}\right]^2} = 1$

which represents an ellipse.

Let e_1 be its eccentricity.

$$\therefore \quad \frac{b^2e^2}{(e+1)^2} = a^2e^2(1-e_1^2)$$

$$\Rightarrow \quad e_1^2 = 1 - \frac{b^2}{a^2(e+1)^2} = 1 - \frac{1-e^2}{(e+1)^2} = 1 - \frac{1-e}{1+e} = \frac{2e}{1+e} \quad \Rightarrow \quad e_1 = \sqrt{\left(\frac{2e}{1+e}\right)}$$

ANSWERS FOR DO YOURSELF

1 : (i) $e = \frac{1}{\sqrt{2}}$ (ii) $\frac{(x-10)^2}{52} + \frac{(y-6)^2}{16} = 1$ (iii) $e = \frac{\sqrt{3}}{2}$; foci $= (1 \pm \sqrt{3}, -1)$; LR $= 1$

2 : (i) $\frac{x^2}{4} + \frac{y^2}{8} = 1$

(ii) C ≡ (–1, 2), length of major axis = 2b = $\sqrt{3}$, length of minor axis = 2a = 1; $e=\sqrt{\frac{2}{3}}$; $f\left(-1,\ 2 \pm \frac{1}{\sqrt{2}}\right)$

(iii) C

3 : (i) On the ellipse (ii) $\frac{1}{2}(a^2 + b^2)$ (iii) $P^2 = a^2\cos^2\theta + b^2\sin^2\theta$

4 : (i) $3y + x \pm \sqrt{97} = 0$ (ii) $7x - 12y = 50$

5 : (i) $4x - 3y = 7$ (ii) abe (iii) -1 (iv) $\frac{\sqrt{3}}{2}$

6 : (i) $\frac{x}{16} + \frac{y}{3} = 1$ (ii) $-\frac{a^4}{b^4}$ (iii) (12, –2)

7 : (i) $-9x + 16y = 25$

8 : (i) 60π (ii) $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{k^2}$

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. If distance between the directrices be thrice the distance between the foci, then eccentricity of ellipse is -

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{4}{5}$

2. If the eccentricity of an ellipse be 5/8 and the distance between its foci be 10, then its latus rectum is -

(A) $\frac{39}{4}$ (B) 12 (C) 15 (D) $\frac{37}{2}$

3. The curve represented by $x = 3(\cos t + \sin t)$, $y = 4(\cos t - \sin t)$, is -

(A) ellipse (B) parabola (C) hyperbola (D) circle

4. If the distance of a point on the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ from the centre is 2, then the eccentric angle is-

(A) $\pi/3$ (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

5. An ellipse having foci at (3, 3) and (–4, 4) and passing through the origin has eccentricity equal to-

(A) $\frac{3}{7}$ (B) $\frac{2}{7}$ (C) $\frac{5}{7}$ (D) $\frac{3}{5}$

6. A tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major & minor axes in points A & B respectively. If C is the centre of the ellipse then the area of the triangle ABC is :

(A) 12 sq. units (B) 24 sq. units (C) 36 sq. units (D) 48 sq. units

7. The equation to the locus of the middle point of the portion of the tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ included between the co-ordinate axes is the curve-

(A) $9x^2 + 16y^2 = 4x^2y^2$ (B) $16x^2 + 9y^2 = 4x^2y^2$

(C) $3x^2 + 4y^2 = 4x^2y^2$ (D) $9x^2 + 16y^2 = x^2y^2$

8. An ellipse is drawn with major and minor axes of lengths 10 and 8 respectively. Using one focus as centre, a circle is drawn that is tangent to the ellipse, with no part of the circle being outside the ellipse. The radius of the circle is-

(A) $\sqrt{3}$ (B) 2 (C) $2\sqrt{2}$ (D) $\sqrt{5}$

9. Which of the following is the common tangent to the ellipses $\frac{x^2}{a^2+b^2}+\frac{y^2}{b^2}=1$ & $\frac{x^2}{a^2}+\frac{y^2}{a^2+b^2}=1$?

(A) $ay = bx + \sqrt{a^4-a^2b^2+b^4}$ (B) $by = ax - \sqrt{a^4+a^2b^2+b^4}$

(C) $ay = bx - \sqrt{a^4+a^2b^2+b^4}$ (D) $by = ax - \sqrt{a^4-a^2b^2+b^4}$

10. Angle between the tangents drawn from point (4, 5) to the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is -

(A) $\frac{\pi}{3}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

11. The point of intersection of the tangents at the point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and its corresponding point Q on the auxiliary circle meet on the line -

(A) $x = a/e$ (B) $x = 0$ (C) $y = 0$ (D) none

12. An ellipse is such that the length of the latus rectum is equal to the sum of the lengths of its semi principal axes. Then -
(A) Ellipse becomes a circle
(B) Ellipse becomes a line segment between the two foci
(C) Ellipse becomes a parabola
(D) none of these

13. The equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the positive end of latus rectum is -
(A) $x + ey + e^2a = 0$ (B) $x - ey - e^3a = 0$ (C) $x - ey - e^2a = 0$ (D) none of these

14. The eccentric angle of the point where the line, $5x - 3y = 8\sqrt{2}$ is a normal to the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ is -
(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\tan^{-1}2$

15. PQ is a double ordinate of the ellipse $x^2 + 9y^2 = 9$, the normal at P meets the diameter through Q at R, then the locus of the mid point of PR is -
(A) a circle (B) a parabola (C) an ellipse (D) a hyperbola

16. The equation of the chord of the ellipse $2x^2 + 5y^2 = 20$ which is bisected at the point (2, 1) is -
(A) $4x + 5y + 13 = 0$ (B) $4x + 5y = 13$ (C) $5x + 4y + 13 = 0$ (D) $4x + 5y = 13$

17. If F_1 & F_2 are the feet of the perpendiculars from the foci S_1 & S_2 of an ellipse $\frac{x^2}{5}+\frac{y^2}{3}=1$ on the tangent at any point P on the ellipse , then $(S_1F_1) . (S_2F_2)$ is equal to
(A) 2 (B) 3 (C) 4 (D) 5

18. If $\tan\theta_1 . \tan\theta_2 = -\frac{a^2}{b^2}$ then the chord joining two points θ_1 & θ_2 on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will subtend a right angle at -
(A) focus (B) centre (C) end of the major axis (D) end of the minor axis

19. The number of values of c such that the straight line $y = 4x + c$ touches the curve $(x^2 / 4) + y^2 = 1$ is -
(A) 0 (B) 1 (C) 2 (D) infinite **[JEE 98]**

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

20. If $x - 2y + k = 0$ is a common tangent to $y^2 = 4x$ & $\frac{x^2}{a^2}+\frac{y^2}{3}=1$ $(a > \sqrt{3})$, then the value of a, k and other common tangent are given by -
(A) $a = 2$ (B) $a = -2$ (C) $x + 2y + 4 = 0$ (D) $k = 4$

21. All ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(0 < b < a)$ has fixed major axis. Tangent at any end point of latus rectum meet at a fixed point which can be -
(A) (a, a) (B) (0, a) (C) (0, –a) (D) (0, 0)

22. Eccentric angle of a point on the ellipse $x^2 + 3y^2 = 6$ at a distance $\sqrt{3}$ units from the centre of the ellipse is -
(A) $\frac{5\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{3\pi}{4}$ (D) $\frac{2\pi}{3}$

23. For the ellipse $9x^2 + 16y^2 - 18x + 32y - 119 = 0$, which of the following is/are true -
(A) centre is (1, –1)
(B) length of major and minor axis are 8 and 6 respectively
(C) $e = \frac{\sqrt{7}}{4}$
(D) foci are $(1 \pm \sqrt{7}, -1)$

24. With respect to the ellipse $4x^2 + 7y^2 = 8$, the correct statement(s) is/are -

(A) length of latus rectum $\frac{8\sqrt{2}}{7}$

(B) the distance between the directrix $4\sqrt{\frac{7}{3}}$

(C) tangent at $\left(\frac{1}{2}, 1\right)$ is $2x + 7y = 8$

(D) Area of Δ formed by foci and one end of minor axis is $\frac{4\sqrt{3}}{7}$

25. On the ellipse, $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are - [JEE 99]

(A) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (B) $\left(-\frac{2}{5}, \frac{1}{5}\right)$ (C) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (D) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

CHECK YOUR GRASP — ANSWER KEY — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A	A	B	C	B	A	B	B	D
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	C	A	B	B	C	B	B	B	C	A,B,C,D
Que.	21	22	23	24	25					
Ans.	B,C	A,B,D	A,B,C,D	A,C,D	B,D					

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. $x - 2y + 4 = 0$ is a common tangent to $y^2 = 4x$ & $\frac{x^2}{4}+\frac{y^2}{b^2}=1$. Then the value of b and the other common tangent are given by -

(A) $b=\sqrt{3}\ ;\ x+2y+4=0$
(B) $b=3\ ;\ x+2y+4=0$
(C) $b=\sqrt{3}\ ;\ x+2y-4=0$
(D) $b=\sqrt{3}\ ;\ x-2y-4=0$

2. The tangent at any point P on a standard ellipse with foci as S & S' meets the tangents at the vertices A & A' in the points V & V', then -

(A) $l(AV).l(A'V')=b^2$
(B) $l(AV).l(A'V')=a^2$
(C) $\angle V'SV=90^\circ$
(D) V'S' VS is a cyclic quadrilateral

3. The area of the rectangle formed by the perpendiculars from the centre of the standard ellipse to the tangent and normal at its point whose eccentric angle is $\pi/4$ is -

(A) $\frac{(a^2-b^2)ab}{a^2+b^2}$
(B) $\frac{(a^2+b^2)ab}{a^2-b^2}$
(C) $\frac{(a^2-b^2)}{ab(a^2+b^2)}$
(D) $\frac{(a^2+b^2)}{(a^2-b^2)ab}$

4. Q is a point on the auxiliary circle of an ellipse. P is the corresponding point on ellipse. N is the foot of perpendicular from focus S, to the tangent of auxiliary circle at Q. Then -

(A) SP = SN
(B) SP = PQ
(C) PN = SP
(D) NQ = SP

5. The line, $lx + my + n = 0$ will cut the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\pi/2$ if -

(A) $x^2 l^2 + b^2 n^2 = 2m^2$
(B) $a^2 m^2 + b^2 l = 2n^2$
(C) $a^2 l^2 + b^2 m^2 = 2n^2$
(D) $a^2 n^2 + b^2 m^2 = 2l$

6. A circle has the same centre as an ellipse & passes through the foci F_1 & F_2 of the ellipse, such that the two curves intersect in 4 points. Let 'P' be any one of their point of intersection. If the major axis of the ellipse is 17 & the area of the triangle PF_1F_2 is 30, then the distance between the foci is -

(A) 11
(B) 12
(C) 13
(D) none

7. The normal at a variable point P on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e meets the axes of the ellipse in Q and R then the locus of the mid-point of QR is a conic with an eccentricity e' such that -

(A) e' is independent of e
(B) e' = 1
(C) e' = e
(D) e' = 1/e

8. The length of the normal (terminated by the major axis) at a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is -

(A) $\frac{b}{a}(r+r_1)$
(B) $\frac{b}{a}|r-r_1|$
(C) $\frac{b}{a}\sqrt{rr_1}$
(D) independent of r, r_1

where r and r_1 are the focal distance of the point.

9. Point 'O' is the centre of the ellipse with major axis AB and minor axis CD. Point F is one focus of the ellipse. If OF = 6 and the diameter of the inscribed circle of triangle OCF is 2, then the product (AB)(CD) is equal to -

(A) 65 (B) 52 (C) 78 (D) none

10. If P is a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci are S and S'. Let $\angle PSS'=\alpha$ and $\angle PS'S=\beta$, then -

(A) PS + PS' = 2a, if a > b

(B) PS + PS' = 2b, if a < b

(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}$

(D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{\sqrt{a^2-b^2}}{b^2}[a-\sqrt{a^2-b^2}]$ when a > b

11. If the chord through the points whose eccentric angles are θ & ϕ on the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the focus, then the value of tan ($\theta/2$) tan ($\phi/2$) is -

(A) $\frac{e+1}{e-1}$ (B) $\frac{e-1}{e+1}$ (C) $\frac{1+e}{1-e}$ (D) $\frac{1-e}{1+e}$

12. If point P(α + 1, α) lies between the ellipse $16x^2 + 9y^2 - 16x = 0$ and its auxiliary circle, then -

(A) [α] = 0

(B) [α] = –1

(C) no such real α exist

(D) [α] = 1

where [.] denotes greatest integer function.

13. If latus rectum of an ellipse $\frac{x^2}{16}+\frac{y}{b^2}=1$ {0< b < 4}, subtend angle 2θ at farthest vertex such that cosecθ = $\sqrt{5}$, then -

(A) $e=\frac{1}{2}$

(B) no such ellipse exist

(C) b = $2\sqrt{3}$

(D) area of Δ formed by LR and nearest vertex is 6 sq. units

14. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) & (x_3, y_3) - **[JEE 99]**

(A) lie on a straight line

(B) lie on an ellipse

(C) lie on a circle

(D) are vertices of a triangle.

BRAIN TEASERS — **ANSWER KEY** — EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A,C,D	A	A	C	C	C	C	A	A,B,C
Que.	11	12	13	14						
Ans.	A,B	A,B	A,C,D	A						

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

FILL IN THE BLANKS

1. The co-ordinates of the mid - point of the variable chord $y = \frac{1}{2}(x+c)$ of the ellipse $4x^2 + 9y^2 = 36$ are _________

2. A triangle ABC right angled at 'A' moves so that it always circumscribes the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The locus of the point 'A' is _________.

3. Atmost _________ normals can be drawn from a point, to an ellipse.

4. Atmost _________ tangents can be drawn from a point, to an ellipse.

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II**.

1.

	Column - I		Column - II
(A)	The minimum and maximum distance of a point (2, 6) from the ellipse are $9x^2 + 8y^2 - 36x - 16y - 28 = 0$	(p)	0
(B)	The minimum and maximum distance of a point $\left(\frac{9}{5},\frac{12}{5}\right)$ from the ellipse $4(3x + 4y)^2 + 9(4x - 3y)^2 = 900$ are	(q)	2
(C)	If E : $2x^2 + y^2 = 2$ and director circle of E is C_1, director circle of C_1 is C_2 director circle of C_2 is C_3 and so on. If r_1, r_2, r_3 .... are the radii of C_1, C_2, C_3 ... respectively then G.M. of r_1^2, r_2^2, r_3^2 is	(r)	6
(D)	Minimum area of the triangle formed by any tangent to the ellipse $x^2 + 4y^2 = 16$ with coordinate axes is	(s)	8

ASSERTION & REASON

These questions contain, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I** : Tangent drawn at a point $P\left(\frac{4\sqrt{5}}{3}, 2\right)$ on the ellipse $9x^2 + 16y^2 = 144$ intersects a straight line $x = \frac{16}{\sqrt{7}}$ at M, then PM subtends a right angle at $(-\sqrt{7}, 0)$

Because

Statement-II : The portion of the tangent to an ellipse between the point of contact and the directrix subtends a right angle at the corresponding focus.

(A) A (B) B (C) C (D) D

2. **Statement-I** : Feet of perpendicular drawn from foci of an ellipse $4x^2 + y^2 = 16$ on the line $2\sqrt{3}x + y = 8$ lie on a circle $x^2 + y^2 = 16$.

Because

Statement-II : If perpendicular are drawn from foci of an ellipse to its any tangent then feet of these perpendicular lie on director circle of the ellipse.

(A) A (B) B (C) C (D) D

3. **Statement-I** : Any chord of the ellipse $x^2 + y^2 + xy = 1$ through (0, 0) is bisected at (0, 0)

Because

Statement-II : The centre of an ellipse is a point through which every chord is bisected.

(A) A (B) B (C) C (D) D

4. **Statement-I** : If $P\left(\frac{3\sqrt{3}}{2}, 1\right)$ is a point on the ellipse $4x^2 + 9y^2 = 36$. Circle drawn AP as diameter touches another circle $x^2 + y^2 = 9$, where $A \equiv (-\sqrt{5}, 0)$

Because

Statement-II : Circle drawn with focal radius as diameter touches the auxilliary circle.

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1 :

An ellipse whose distance between foci S and S' is 4 units is inscribed in the triangle ABC touching the sides AB, AC and BC at P, Q and R. If centre of ellipse is at origin and major axis along x-axis, SP + S'P = 6.

On the basis of above information, answer the following questions :

1. If $\angle BAC = 90$, then locus of point A is -

(A) $x^2 + y^2 = 12$ (B) $x^2 + y^2 = 4$ (C) $x^2 + y^2 = 14$ (D) none of these

2. If chord PQ subtends 90 angle at centre of ellipse, then locus of A is -

(A) $25x^2 + 81y^2 = 620$ (B) $25x^2 + 81y^2 = 630$ (C) $9x^2 + 16y^2 = 25$ (D) none of these

3. If difference of eccentric angles of points P and Q is 60 , then locus of A is -

(A) $16x^2 + 9y^2 = 144$ (B) $16x^2 + 45y^2 = 576$ (C) $5x^2 + 9y^2 = 60$ (D) $5x^2 + 9y^2 = 15$

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE-3

- *Fill in the Blanks*

1. $-\frac{9}{25}c, \frac{8}{25}c$ 2. $x^2 + y^2 = a^2 + b^2$, director circle 3. 4 4. 2

- *Match the Column*

1. (A) → (q,s) ; (B) → (p,r); (C) → (r); (D) → (s)

- *Assertion & Reason*

1. D 2. C 3. A 4. A

- *Comprehension Based Questions*

Comprehension # 1 : 1. C 2. B 3. C

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

1. Find the equation to the ellipse, whose focus is the point (–1, 1), whose directrix is the straight line $x - y + 3 = 0$ and whose eccentricity is $\frac{1}{2}$.

2. Find the latus rectum, the eccentricity and the coordinates of the foci, of the ellipse

(a) $x^2 + 3y^2 = a^2,\ a > 0$ (b) $5x^2 + 4y^2 = 1$

3. Find the eccentricity of an ellipse in which distance between their foci is 10 and that of focus and corresponding directrix is 15.

4. If focus and corresponding directrix of an ellipse are (3, 4) and $x + y - 1 = 0$ and eccentricity is $\frac{1}{2}$ then find the co-ordinates of extremities of major axis.

5. An ellipse passes through the points (– 3 , 1) & (2 , –2) & its principal axis are along the coordinate axes in order. Find its equation.

6. Find the latus rectum, eccentricity, coordinates of the foci, coordinates of the vertices, the length of the axes and the centre of the ellipse $4x^2 + 9y^2 - 8x - 36y + 4 = 0$.

7. Find the set of value(s) of α for which the point $\left(7-\frac{5}{4}\alpha, \alpha\right)$ lies inside the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$.

8. Find the condition so that the line $px + qy = r$ intersects the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in points whose eccentric angles differ by $\frac{\pi}{4}$.

9. Find the equations of the lines with equal intercepts on the axes & which touch the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

10. The tangent at the point α on a standard ellipse meets the auxiliary circle in two points which subtends a right angle at the centre. Show that the eccentricity of the ellipse is $(1 + \sin\alpha)^{-1/2}$.

11. Find the equation of tangents to the ellipse $\frac{x^2}{50}+\frac{y^2}{32}=1$ which passes through a point (15, –4).

12. ABC is an isosceles triangle with its base BC twice its altitude. A point P moves within the triangle such that the square of its distance from BC is half the area of rectangle contained by its distances from the two sides . Show that the locus of P is an ellipse with eccentricity $\sqrt{\frac{2}{3}}$ passing through B & C.

13. 'O' is the origin & also the centre of two concentric circles having radii of the inner & the outer circle as 'a' & 'b' respectively . A line OPQ is drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y-axis & QR is drawn parallel to the x– axis . Prove that the locus of R is an ellipse touching the two circles. If the foci of this ellipse lie on the inner circle , find the ratio of inner : outer radii & find also the eccentricity of the ellipse.

14. If tangent drawn at a point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is same as the normal drawn at a point $\left(\sqrt{5}\cos\phi, 2\sin\phi\right)$ on the ellipse $4x^2 + 5y^2 = 20$, then find the values of t & ϕ.

15. The tangent and normal to the ellipse $x^2 + 4y^2 = 4$ at a point $P(\theta)$ on it meet the major axis in Q and R respectively. If QR = 2, show that the eccentric angle θ of P is given by $\cos\theta = \pm (2/3)$.

16. If the normal at a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of semi axes a , b & centre C cuts the major & minor axes at G & g , show that $a^2.(CG)^2 + b^2.(Cg)^2 = (a^2 - b^2)^2$. Also prove that $CG = e^2CN$, where PN is the ordinate of P. (N is foot of perpendicular from P on its major axis.)

17. A ray emanating from the point (–4, 0) is incident on the ellipse $9x^2 + 25y^2 = 225$ at the point P with abscissa 3. Find the equation of the reflected ray after first reflection.

18. Find the locus of the point the chord of contact of the tangent drawn from which to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches the circle $x^2 + y^2 = c^2$, where c < b < a.

19. If 3x + 4y = 12 intersect the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ at P and Q, then find the point of intersection of tangents at P and Q.

20. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P & Q. Prove that the tangents at P & Q of the ellipse $x^2 + 2y^2 = 6$ are at right angles.

CONCEPTUAL SUBJECTIVE EXERCISE — **ANSWER KEY** — EXERCISE-4(A)

1. $7x^2 + 2xy + 7y^2 + 10x - 10y + 7 = 0$ 2. (a) $\frac{2a}{3};\frac{1}{3}\sqrt{6};\left(\pm\frac{a}{3}\sqrt{6},0\right)$ (b) $\frac{4}{5};\frac{1}{5}\sqrt{5};\left(0,\pm\frac{1}{10}\sqrt{5}\right)$

3. $\left(e=\frac{1}{2}\right)$ 4. ((2, 3) & (6, 7)) 5. 3x + 5y = 32

6. $\frac{8}{3},\frac{\sqrt{5}}{3};\ \left(1\pm\sqrt{5},2\right);\left(-2,2\right)$ and (4, 2); 6 and 4; (1, 2) 7. $\left(\frac{12}{5},\frac{16}{5}\right)$

8. $a^2p^2 + b^2q^2 = r^2\sec^2\frac{\pi}{8} = (4 - 2\sqrt{2})\,r^2$ 9. x + y – 5 = 0 , x + y + 5 = 0

11. 4x + 5y = 40, 4x – 35y = 200

13. $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$ 14. $\phi=\pi-\tan^{-1}2,\ t=-\frac{1}{\sqrt{5}};\ \phi=\pi+\tan^{-1}2,\ t=\frac{1}{\sqrt{5}};\ \phi=\frac{\pi}{2},\frac{3\pi}{2}\ \ t=0$

17. 12x + 5y = 48; 12x – 5y =48 18. $\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{c^2}$ 19. $\left(\frac{25}{4},\frac{16}{3}\right)$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. The tangent at any point P of a circle $x^2 + y^2 = a^2$ meets the tangent at a fixed point A (a, 0) in T and T is joined to B, the other end of the diameter through A. Prove that the locus of the intersection of AP and BT is an ellipse whose eccentricity is $\frac{1}{\sqrt{2}}$.

2. The tangent at $P\left(4\cos\theta, \frac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x - 15 = 0$. Find θ. Find also the equation to the common tangent.

3. Common tangents are drawn to the parabola $y^2 = 4x$ & the ellipse $3x^2 + 8y^2 = 48$ touching the parabola at A & B and the ellipse at C & D. Find the area of the quadrilateral.

4. Find the equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$.

5. Prove that the length of the focal chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which is inclined to the major axis at angle θ is $\frac{2ab^2}{a^2\sin^2\theta + b^2\cos^2\theta}$.

6. The tangent at a point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersects the major axis in T & N is the foot of the perpendicular from P to the same axis. Show that the circle on NT as diameter intersects the auxiliary circle orthogonally.

7. The tangents from (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ intersect at right angles. Show that the normals at the points of contact meet on the line $\frac{y}{y_1} = \frac{x}{x_1}$.

8. If the normals at the points P, Q, R with eccentric angles α, β, γ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are concurrent, then show that $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin 2\alpha \\ \sin\beta & \cos\beta & \sin 2\beta \\ \sin\gamma & \cos\gamma & \sin 2\gamma \end{vmatrix} = 0$.

9. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the tangent drawn at a point P on the ellipse. If F_1 & F_2 are the two foci of the ellipse, then show that $(PF_1 - PF_2)^2 = 4a^2\left[1 - \frac{b^2}{d^2}\right]$

10. Consider the family of circles, $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of the family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at A & B, then find the equation of the locus of the mid-point of AB. [JEE 99]

BRAIN STORMING SUBJECTIVE EXERCISE **ANSWER KEY** EXERCISE-4(B)

2. $\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$; $4x \pm \sqrt{33}\,y = 32$ 3. $55\sqrt{2}$ sq. units 4. $(x-1)^2 + y^2 = \frac{11}{3}$

10. $25y^2 + 4x^2 = 4x^2y^2$

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. If distance between the foci of an ellipse is equal to its minor axis, then eccentricity of the ellipse is- [AIEEE-2002]

(1) $e = \frac{1}{\sqrt{2}}$ (2) $e = \frac{1}{\sqrt{3}}$ (3) $e = \frac{1}{\sqrt{4}}$ (4) $e = \frac{1}{\sqrt{6}}$

2. The equation of an ellipse, whose major axis = 8 and eccentricity = 1/2 is- (a > b) [AIEEE-2002]

(1) $3x^2 + 4y^2 = 12$ (2) $3x^2 + 4y^2 = 48$ (3) $4x^2 + 3y^2 = 48$ (4) $3x^2 + 9y^2 = 12$

3. The eccentricity of an ellipse, with its centre at the origin, is 1/2. If one of the directirices is x = 4, then the equation of the ellipse is- [AIEEE-2004]

(1) $3x^2 + 4y^2 = 1$ (2) $3x^2 + 4y^2 = 12$ (3) $4x^2 + 3y^2 = 12$ (4) $4x^2 + 3y^2 = 1$

4. An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is- [AIEEE-2005, IIT-1997]

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{3}}$

5. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is- [AIEEE-2006]

(1) $\frac{1}{2}$ (2) $\frac{4}{5}$ (3) $\frac{1}{\sqrt{5}}$ (4) $\frac{3}{5}$

6. A focus of an ellipse is at the origin. The directrix is the line x = 4 and the eccentricity is 1/2. Then the length of the semi-major axis is- [AIEEE-2008]

(1) 8/3 (2) 2/3 (3) 4/3 (4) 5/3

7. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is :- [AIEEE-2009]

(1) $4x^2 + 48y^2 = 48$ (2) $4x^2 + 64y^2 = 48$ (3) $x^2 + 16y^2 = 16$ (4) $x^2 + 12y^2 = 16$

8. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{2/5}$ is :- [AIEEE-2011]

(1) $3x^2 + 5y^2 - 15 = 0$ (2) $5x^2 + 3y^2 - 32 = 0$ (3) $3x^2 + 5y^2 - 32 = 0$ (4) $5x^2 + 3y^2 - 48 = 0$

9. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : [AIEEE-2012]

(1) $x^2 + 4y^2 = 16$ (2) $4x^2 + y^2 = 4$ (3) $x^2 + 4y^2 = 8$ (4) $4x^2 + y^2 = 8$

10. **Statement–1** : An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement–2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$. [AIEEE-2012]

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

11. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having centre at (0, 3) is : [JEE (Main)-2013]

(1) $x^2 + y^2 - 6y - 7 = 0$ (2) $x^2 + y^2 - 6y + 7 = 0$ (3) $x^2 + y^2 - 6y - 5 = 0$ (4) $x^2 + y^2 - 6y + 5 = 0$

PREVIOUS YEARS QUESTIONS — ANSWER KEY — EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10	11
Ans	1	2	2	1	4	1	4	3	1	3	1

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from A, B, C to the major axis of the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, (a > b) meet the ellipse respectively at P,Q,R so that P, Q,R lie on the same side of the major axis as A, B,C respectively . Prove that the normals to the ellipse drawn at the points P, Q and R are concurrent. **[JEE 2000 (Mains) 7M]**

2. Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C. **[JEE 2001 (Mains) 5M]**

3. Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. **[JEE 2002 (Mains) 5M]**

4. Tangent is drawn to ellipse $\frac{x^2}{27}+y^2=1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ (where $\theta \in (0, \pi/2)$). Then the value of θ, such that sum of intercepts on axes made by this tangent is least is - **[JEE 2003 (Screening)]**

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{8}$ (D) $\frac{\pi}{4}$

5. The area of the quadrilateral formed by the tangents at the end points of the latus rectum of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is -

(A) 27/4 sq. units (B) 9 sq. units (C) 27/2 sq. units (D) 27 sq. units

[JEE 2003 (Screening)]

6. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line x + y = 7, is as small as possible.

[JEE 2003 (Main) 2M out of 60]

7. Locus of the mid points of the segments which are tangents to the ellipse $\frac{1}{2}x^2+y^2=1$ and which are intercepted between the coordinate axes is - **[JEE 2004 (Screening)]**

(A) $\frac{1}{2}x^2+\frac{1}{4}y^2=1$ (B) $\frac{1}{4}x^2+\frac{1}{2}y^2=1$ (C) $\frac{1}{3x^2}+\frac{1}{4y^2}=1$ (D) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$

8. The minimum area of triangle formed by tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and coordinate axes -

(A) ab (B) $\frac{a^2+b^2}{2}$ (C) $\frac{(a+b)^2}{2}$ (D) $\frac{a^2+ab+b^2}{3}$

[JEE 2005 (Screening)]

9. Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$. Also find the length of the intercept of the tangent between the coordinate axes.

[JEE 2005 (Mains) 4M out of 60]

10. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2+4y^2=4$. The equations of parabolas with latus rectum PQ are - **[JEE 2008, 4M]**

(A) $x^2 + 2\sqrt{3}\,y = 3 + \sqrt{3}$ (B) $x^2 - 2\sqrt{3}\,y = 3 + \sqrt{3}$

(C) $x^2 + 2\sqrt{3}\,y = 3 - \sqrt{3}$ (D) $x^2 - 2\sqrt{3}\,y = 3 - \sqrt{3}$

11. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is :- **[JEE 2009, 3M, –1M]**

(A) $\frac{31}{10}$ (B) $\frac{29}{10}$ (C) $\frac{21}{10}$ (D) $\frac{27}{10}$

12. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points - **[JEE 2009, 3M, –1M]**

(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2} \pm\frac{\sqrt{19}}{4}\right)$ (C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Paragraph for Question 13 to 15 **[JEE 10, (3M each), –1M]**

Tangents are drawn from the point P(3, 4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B.

13. The coordinates of A and B are

(A) (3, 0) and (0, 2)

(B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and (0, 2)

(D) (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

14. The orthocenter of the triangle PAB is

(A) $\left(5, \frac{8}{7}\right)$ (B) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (C) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (D) $\left(\frac{8}{25}, \frac{7}{5}\right)$

15. The equation of the locus of the point whose distances from the point P and the line AB are equal, is

(A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

(B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$

(D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

16. The ellipse $E_1 : \frac{x^2}{9} + \frac{x^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0,4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is - **[JEE 2012, 3M, –1M]**

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

17. A vertical line passing through the point (h,0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$ **[JEE-Advanced 2013, 4, (–1)M]**

PREVIOUS YEARS QUESTIONS	**ANSWER KEY**	EXERCISE-5 [B]

2. Locus is an ellipse with foci as the centres of the circles C_1 and C_2. **4.** B **5.** D **6.** (2,1)

7. D **8.** A **9.** $y = -\frac{2}{\sqrt{3}}x + 4\sqrt{\frac{7}{3}}$; $\frac{14}{\sqrt{3}}$ **10.** B,C **11.** D **12.** C **13.** D **14.** C **15.** A

16. C **17.** 9